THERMAL RADIATION ON AN UNSTEADY MHD DOUBLE DIFFUSIVE AND CHEMICALLY REACTIVE FLOW PAST AN INFINITE VERTICAL POROUS PLATE WITH HEAT SOURCE.

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ABSTRACT

An unsteady two dimensional flow of a laminar, viscous, electrically conducting chemically reacting viscous dissipative and heat absorbing fluid past a semi infinite vertically permeable moving plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field in the presence of thermal and concentration buoyancy effects was considered. The plate is subjected to a constant normal suction/ injection velocity. The governing equation for this investigation, which was based on the balanced mass, linear momentum, energy and species concentration, were solved analytically using perturbation technique.

INTRODUCTION

The study of Magneto hydrodynamics (MHD) flow of different kinds of fluids in different medium has attracted the attention of many researchers

Ravikumar et al (2013) worked on MHD double diffusive and chemically reactive flow through porous medium bounded by two vertical plates, considering a steady state and using Reynolds number parameter in solving the non- dimensional equation resulting into perturbation technique the numerical computation of the result were obtained and were used to analyse the result graphically. Manta and Venkata (2014) investigate on the thermal radiation effect on an unsteady

MHD free convective chemically reacting viscous dissipative fluid past an infinite vertical moving porous plate with heat source. Anuradha and Priyadharshini (2014) worked on heat and mass transfer on unsteady MHD free convective flow pass a semi-infinite vertical plate with soret effect, using perturbation method in solving the equation subjected to the constant radiation term. Ibrahim et al (2014) studied the effect of chemical reaction and radiation absorption on the unsteady MHD free convective flow past a semi-infinite vertical permeable moving plate with heat source and suction.

FORMULATION OF PROBLEM.

We consider unsteady two dimensional flow of a laminar, viscous, electrically conducting chemically reacting viscous dissipative and heat absorbing fluid past an infinite vertical permeable moving plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field in the presence of thermal and concentration buoyancy effects. thakur and krishua (2014). In the Cartesian coordinate system, let x' be taken along the plate and y' be perpendicular to the plate, since the plate is infinite in x'- direction. The wall is maintained at constant temperature T'_ω and concentration C'_ω higher than the ambient temperature T'_∞ and concentration C'_∞ respectively. A uniform magnetic field of magnitude β_0 is applied normal to the plate. The transverse applied magnetic field and magnetic Reynolds' number are assumed to be very small, so that the induced magnetic field is negligible.

A uniform magnetic field acts perpendicular to the porous surface which absorbs the fluid with a suction velocity varying with time. The plate is assumed to move with a constant velocity in the direction of the fluid flow. The fluid properties are assumed to be constant except that the influence of density variation with temperature has been considered only in the body.

ISSN 2229-5518

Due to the infinite plane surface assumption, the flow variables are voltage which implies the absence of an electric field. The fluid has constant viscosity and constant thermal conductivity. The fluid is considered to be gray-absorbing emitting radiation but non-scattering medium and the Rosseland's approximation is used to describe the radiative heat flux. It is considered to be negligible in x' direction as compared in y' direction.

The concentration of diffusing species is very small in comparism to other chemical species, the concentration of species far from the wall C'_∞ is infinitesimally small and hence the Soret and Dufour effect are neglected.

Considering the assumptions made the governing equations can be rewritten as follows.

Continuity equation

$$\frac{\partial v}{\partial y} = 0 \qquad 1$$

Which is satisfied with $v = v_o$ constant suction / injection

Momentum equation.

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = v\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty) - \frac{VU'}{K} - \frac{\sigma B_0 u'}{\rho} \qquad 2$$

Energy equation.

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_P}\frac{\partial^2 T'}{\partial y'^2} + \frac{Q_0}{\rho C_P}(T' - T'_\infty) - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y'} \qquad 3$$

Species equation

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} + D_1\frac{\partial^2 T'}{\partial y'^2} - K_1(C' - C'_\infty). \qquad 4$$

We introduced the following non dimensional quantities.

$$y = \frac{y'V_0}{v}, \quad t = \frac{t'V_0^2}{v}, \quad u = \frac{u'}{u_0}, \quad \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty},$$

$$\phi = \frac{C' - C'_\infty}{C'_\infty - C'_\infty}, \quad U_P = \frac{U'_P}{V_0}, \quad G_r = \frac{g\beta v(T'_\omega - T'_\infty)}{U_0 V_0^2}, \quad G_C = \frac{g\beta' v(C'_\omega - C'_\infty)}{U_0 V_0^2},$$

$$P_r = \frac{\nu \rho C_P}{\kappa}, \qquad S_C = \frac{\nu}{D}, \qquad M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \qquad S = \frac{\nu Q_0}{\rho C_{PV_0^2}} \qquad 5 \qquad ,$$

$$K = \frac{K' V_0^2}{V^2}\ , \qquad K_r = \frac{K_r' \nu}{V_0^2}, \qquad R = \frac{\kappa K'}{4\sigma T_h'^3}, \qquad E_C = \frac{V_0^2}{C_P(T_\omega' - T_\infty')}$$

The governing equations for momentum, energy and species equations are:

$$\frac{\partial \mathrm{u}}{\partial t} + \frac{v' \partial u}{v_0 \partial y} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m\phi - \left(M + \frac{1}{K}\right)u \qquad 6$$

$$\frac{\partial \theta}{\partial t} + \frac{v'}{v_0}\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\left(1 + \frac{4R}{3}\right)\frac{\partial^2 \theta}{\partial y^2} - S\theta \qquad 7$$

$$\frac{\partial \phi}{\partial t} + \frac{v'}{v_0}\frac{\partial \phi}{\partial y} = \frac{1}{S_C}\frac{\partial^2 \phi}{\partial t^2} + S_0\frac{\partial \theta}{\partial y} - K_r\phi \qquad 8$$

$$v' = -v_0(1 + \varepsilon e^{nt}).$$

Suction velocity in the plate.

Where v_0 is scale of suction velocity.

ε is the value less than a unit.

Negative sign indicate that the suction is towards the plate.

Substituting this $\frac{v'}{v_0} = -(1 + \varepsilon e^{nt})$ into equation 6, 7, and 8. We obtain

$$\frac{\partial \mathrm{u}}{\partial t} - (1 + \varepsilon e^{nt})\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m\phi - \left(M + \frac{1}{K}\right)u \qquad 9$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon e^{nt})\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\left(1 + \frac{4R}{3}\right)\frac{\partial^2 \theta}{\partial y^2} - S\theta \qquad 10$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon e^{nt})\frac{\partial \phi}{\partial y} = \frac{1}{S_C}\frac{\partial^2 \phi}{\partial t^2} + S_0\frac{\partial \theta}{\partial y} - K_r\phi \qquad 11$$

In order to solve the partial differential equations, subjected to the boundary condition; we assumed that velocity, temperature and concentration in a series expansion in power of ε where $\varepsilon \ll 1$ as

$$u(y,t) = u_0(y) + \ \varepsilon u_1 e^{nt} + 0(\varepsilon^2) + \ldots. \qquad 12$$

$$\theta(y,t) = \theta_0(y) + \ \varepsilon \theta_1 e^{nt} + 0(\varepsilon^2) + \ldots. \qquad 13$$

$$\phi(y,t) = \phi_0(y) + \ \varepsilon \phi, e^{nt} + 0(\varepsilon^2) + \ldots. \qquad 14$$

ISSN 2229-5518

Substituting equation (12) to (14) into equation (9) to (11) and equation the coefficient of similar powers of epsilon and neglecting the higher powers of epsilon, we obtain the following ordinary differential equations for $\left(U_{0,}\ \ \theta_{0,}\ \ \emptyset_0\right),\ and\ (U_{1,}\ \ \theta_{1,}\ \ \emptyset_1)$

$$U_0'' + U_0' - \left(M + {}^1/_k\right)U_0 = -Gr\theta_0 - Gm\emptyset_0 \quad 15$$

$$\left(1 + {}^{4R}/_3\right)\theta_0'' + Pr\theta_0' - SPr\theta_0 = 0 \quad 16$$

$$\emptyset_0'' + Sc\,\emptyset_0' - KrSc\,\emptyset_0 = -Sc\,So\,\theta'' \quad 17$$

$$U_1'' + U_,' - \left(M + n + {}^1/_k\right)U_1 = -Gr\theta_1 - Gm\emptyset_1 - \left(M + {}^1/_k\right)U_0 - U_0' \quad 18$$

$$\left(1 + {}^{4R}/_3\right)\theta_1'' + P_r\theta_1' - \mathrm{P_r}(s+n)\theta_1 = -\theta_0' \quad 19$$

$$\emptyset_1'' + Sc\,\emptyset_1' - (Kr+n)\,Sc\phi_1 = -Sc\,\emptyset_0' - S_0\,S_c\,\theta_,'' \quad 20$$

Subjected to the boundary condition

U_0'=0, $\emptyset_0'$=1, θ_0'=1, as y=0 21

$U_1'' \to 0$ $\emptyset_1'' \to 1$ $\theta_1'' \to 1$ as y$\to \infty$

Solving equation (15) to (20) with respect to the boundary condition (21), we obtain the following solution.

$$\theta_0 = C_1e^{m_2y}$$

$$\theta_1 = C_3e^{a_2y} + C_4e^{m_2y}$$

$$\emptyset_0 = C_6e^{b_2y} + C_7e^{m_2y}$$

$$\emptyset_1 = C_9e^{d_2y} + C_{10}e^{b_2y} + C_{11}e^{a_2y} + C_{12}e^{m_2y}$$

$$U_0 = C_{14}e^{r_2y} + C_{15}e^{b_2y} + C_{16}e^{m_2y}$$

$$U_1 = C_{18}e^{z_2y} + C_{19}e^{a_2y} + C_{20}e^{m_2y} + C_{21}e^{d_2y} + C_{22}e^{b_2y} + C_{23}e^{r_2y}$$

Where

$$C_1 = 1$$

$$C_3 = 1 - C_4$$

$$C_4 = \frac{M_2}{nP_r}$$

$$C_6 = 1 - C_7$$

ISSN 2229-5518

$$C_7 = \frac{-S_c S_0 M_2^2}{m_2^2+S_c m_2-K_r S_c}$$

$$C_9 = 1-[C_{10}+C_{11}+C_{12}]$$

$$C_{10} = \frac{-S_c b_2 C_6}{b_2^2+S_c b_2-(k_r+n)S_c}$$

$$C_{11} = \frac{-S_0 S_c C_3 a_2^2}{a_2^2+S_c a_2-(k_r+n)S_c}$$

$$C_{12} = \frac{-[S_c m_2 C_7+S_0 S_c C_4 m_2^2}{m_2^2+S_c m_2-(k_r+n)S_c}$$

$$C_{14} = -[C_{15}+C_{16}]$$

$$C_{15} = \frac{-G_m C_6}{b_2^2+b_2-(M+{}^1/_k)}$$

$$C_{16} = \frac{-[G_r+C_7 G_m]}{m_2^2+m_2-(M+{}^1/_k)}$$

$$C_{18} = -[C_{19}+C_{20}+C_{21}+C_{22}+C_{23}]$$

$$C_{19} = \frac{-[G_r C_3+G_m C_{11}}{a_2^2+a_2-(M+n+{}^1/_k)}$$

$$C_{20} = \frac{-(G_r C_4+G_m C_{12}+(M+{}^1/_k+M_2))}{m_2^2+m_2-(M+n+{}^1/_k)}$$

$$C_{21} = \frac{-G_m C_9}{d_2^2+d_2-(M+n+{}^1/_k)}$$

$$C_{22} = \frac{-(G_m C_{10}+(M+{}^1/_k+b_2)C_{15}}{b_2^2+b_2-(M+n+{}^1/_k)}$$

$$C_{23} = \frac{-(M+{}^1/_k+r_2)C_{14}}{r_2^2+r_2-(M+n+{}^1/_k)}$$

And

$$m_2 = \frac{-P_r-\sqrt{Pr^2+4sPr(1+{}^{4R}/_3)}}{2(1+{}^{4R}/_3)}$$

$$a_2 = \frac{-P_r-\sqrt{Pr^2+4(1+{}^{4R}/_3)P_r(s+n)}}{2(1+{}^{4R}/_3)}$$

$$b_2 = {}^1/_2\,(-sc-\sqrt{sc^2+4k_r s_c})$$

$$d_2 = {}^1/_2\,(-sc-\sqrt{sc^2+4(k_r+n)s_c})$$

$$r_2 = {}^1/_2\,(-1-\sqrt{1+4(M+{}^1/_K)\,s_c})$$

$$z_2 = {}^1/_2\;(-1-\sqrt{1+4(M+n+{}^1/_K)})$$

International Journal of Scientific & Engineering Research, Volume 6, Issue 11, November-2015 1554
ISSN 2229-5518

In view of above solution, the velocity, concentration and temperature distribution in boundary layer becomes

$U(y,t) = U_0(y) + \varepsilon e^{nt} U_1(y)$

$\emptyset(y,t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y)$

$\theta(y,t) = \emptyset_0(y) + \varepsilon e^{nt} \emptyset_1(y)$

$U(y,t) = c_{14} e^{r_2 y} + c_{15} e^{b_2 y} + c_{16} e^{m_2 y} + \varepsilon e^{nt}(c_{18} e^{r_2 y} + c_{19} e^{a_2 y} + c_{20} e^{m_2 y} + c_{21} e^{d_2 y} + c_{22} e^{b_2 y} + c_{23} e^{r_2 y})$

$\emptyset(y,t) = c_6 e^{b_2 y} + c_7 e^{m_2 y} + \varepsilon e^{nt}(c_9 e^{d_2 y} + c_{10} e^{b_2 y} + c_{11} e^{a_2 y} + c_{12} e^{m_2 y})$

$\theta(y,t) = c_1 e^{m_2 y} + \varepsilon e^{nt}(c_3 e^{a_2 y} + c_4 e^{m_2 y})$

Calculation for physical quantities

SKIN FRICTION

The skin friction due to local wall shear is given by

$c_f = \left(\frac{du}{dy}\right)_{y=0}$

$c_2 c_{14} + b_2 c_{15} + m_2 c_{16} + \varepsilon e^{nt}(z_2 c_{18} + a_2 c_{19} + m_2 c_{20} + d_2 c_{21} + b_2 c_{22} + r_2 c_{22})$

SHERWOOD NUMBER

Mass transfer coefficient (sh) at the slave in terms of amplitude and phase is given by

$sh = -\left(\frac{d\phi}{dy}\right)_{y=0}$

$= b_2 c_6 + m_2 c_7 + \varepsilon e^{nt}(d_2 c_9 + b_2 c_{10} + a_2 c_{11} + m_2 c_{12})$

NUSSELT NUMBER

The rate heat transfer in terms of nusselt number *Nu* is given by

$Nu = -(1 + {}^{4r}/_{3})(\frac{d\theta}{dy})_{y=0}$

$= -(1 + {}^{4R}/_{3})\{(m_2 c_1 + \varepsilon e^{nt}(a_2 c_3 + m_2 c_4)\}$

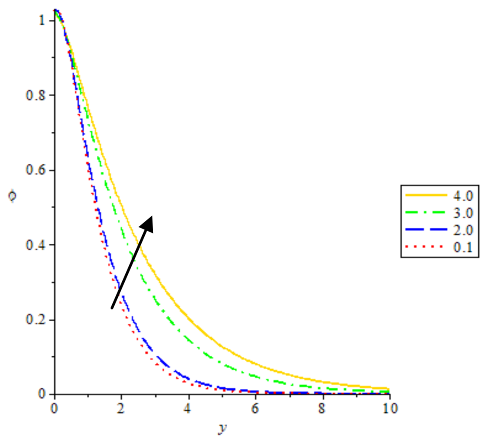

Figure 4.1: The graph of concentration profile for different values of R.

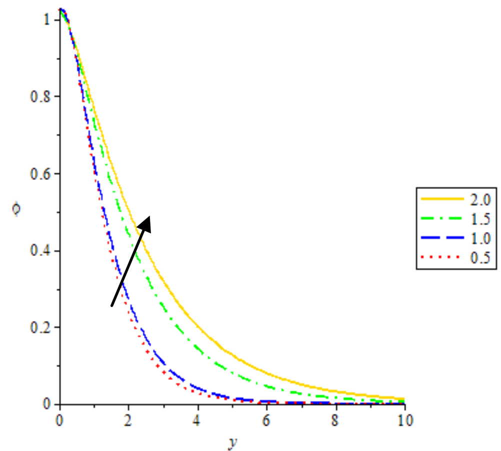

Figure 4.2: The graph of concentration profile for different values of S

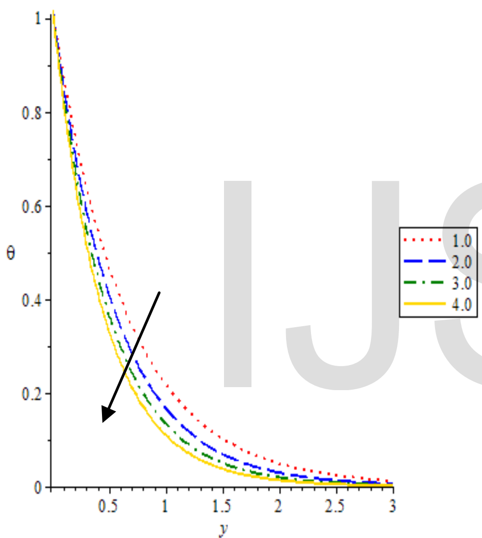

Figure 4.5: The graph of temperature profile for different values of R

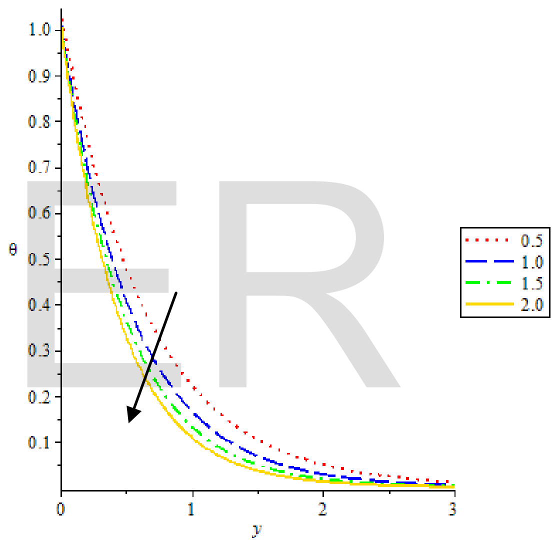

Figure 4.6: The graph of temperature profile for different values of S

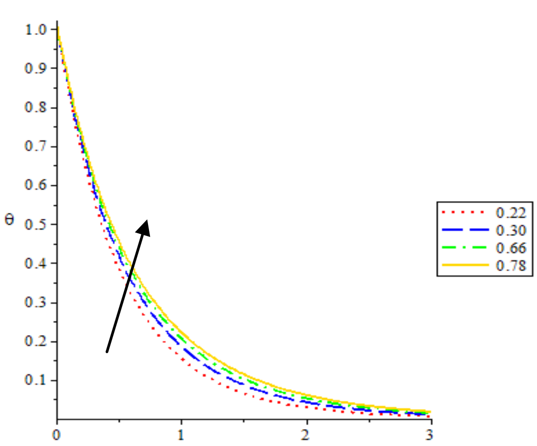

Figure 4.7: The graph of temperature profile for different values of Sc

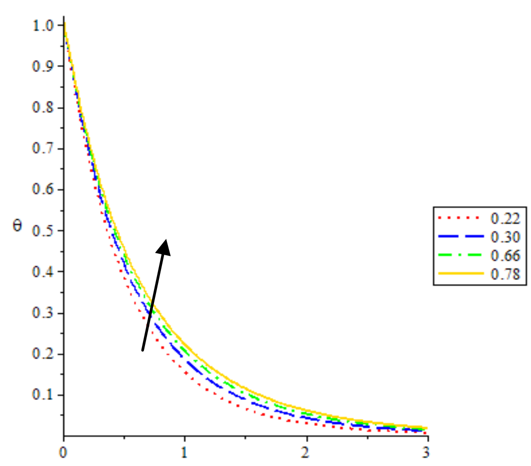

Figure 4.7: The graph of temperature profile for different values of Sc

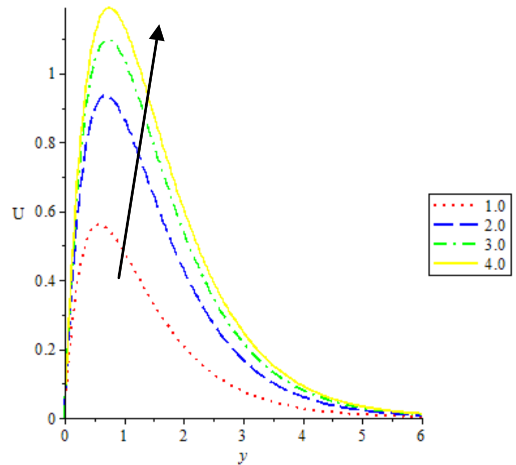

Figure 4.10:The graph of velocity profile for different valuesof R

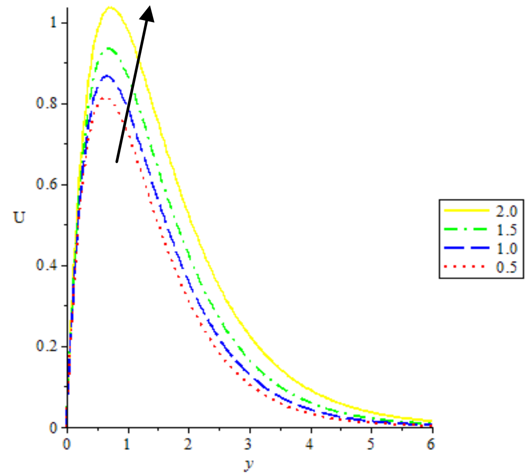

Figure 4.11: The graph of velocity profile for different values of S

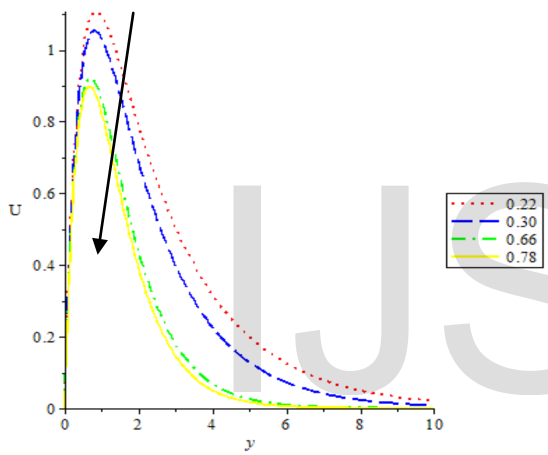

Figure 4.12: The graph of velocity profile for different values of Sc

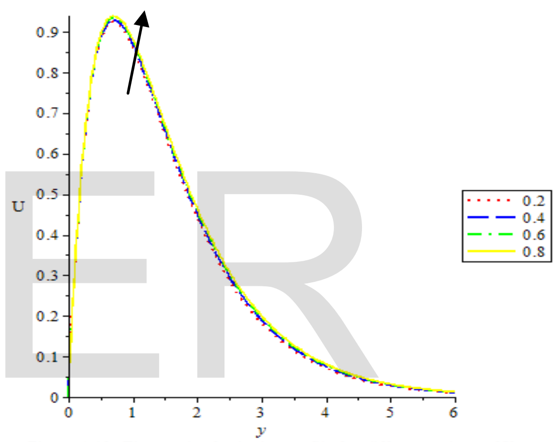

Figure 4.13: The graph of velocity profile for different values of K

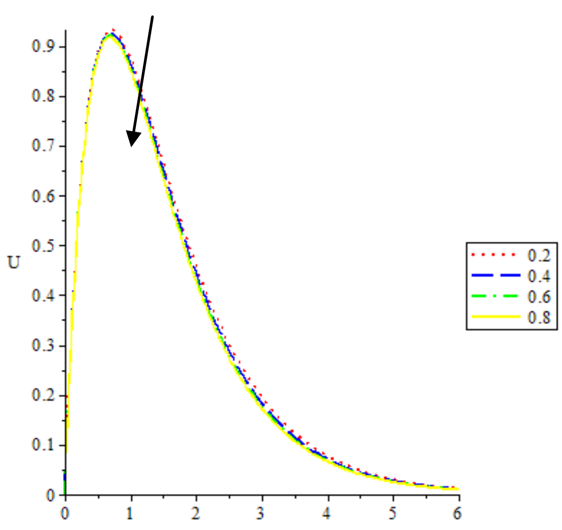

Figure 4.14: The graph of velocity profile for different values of Kr

Discussion of result.

In order to get a clear insight of the physical problem numerical, results are deployed with the help pf graphs. This enables us to carry out the numerical calculation for the distribution of the concentration, temperature and velocity across the boundary layer for various of the parameters. In the present study we have chosen A=0.5, T=1.0, n=0.1, ε=0.02, G_r=5.0, G_m=2.0, M=1.0, P_r=0.71, while R, S, $S_{0,}$ K, Kr are varied over a range, which are listed in the figures.

Concentration profile for different values of radiation terms are described in figure 4.1 it is observed that increase in radiation parameter leads to increase in concentration profile. Hence thermal radiation enhances convention flow.

Figure 4.2 shows the concentration profile for different values for heat source S, we observed that the heat is generated as the buoyancy force increases which induced the flow rate to increase and hence the concentration profile also increase. Figure 4.3 shows the effect of increasing values of Schmidt number (Sc) results in a decrease in concentration distribution, because the smaller values of Sc are equivalent to the increase in the chemical molecular diffusivity.

The effect of radiation parameter (R) on the temperature profiles are presented in figure 4.5. From this figure we observe that, as the value of R increases the temperature profile decreases, with an increasing in the thermal boundary layer thickness. The effect of heat source on temperature profile is observed in figure 4.6. It is seen in the figure that the temperature profile increases with increasing in heat source parameter.

Figure 4.7 shows the effect of Schmidt number Sc on temperature profile. It is observed that increase in values of Sc leads to increase in the temperature profile. There are no changes in the temperature profile of the permeability (K) and chemical permeability (Kr) as shown in figure 4.8 and 4.9 respectively.

ISSN 2229-5518

Figure 4.10 shows the effect of thermal radiation (R) on the velocity profile. It is observed that increase in R leads to increase in velocity profile. Similarly it is observed, as shown in figure 4.11 that increase in heat source (S) results in the increase in the velocity profile.

Figure 4.12 shows the effect of Schmidt number Sc on the velocity profile. We can see, in the figure that increase in values of Sc leads to decrease in the velocity profile. Figure 4.13 shows the effect of chemical permeability (Kr) on the velocity profile, it is observed that increase in Kr leads to decrease in velocity profile.

Figure 4.13 shows the effect of permeability (K) on the velocity profile. It is observed that increase in the values of K leads to increase in velocity profile. This result could be very important in deciding the applicability of enhancing oil reservoir engineering.

CONCLUSION

In this paper we have investigated thermal radition on an unsteady MHD double diffusive and chemically reactive flow past an infinite porous plate with heat source. From the present study we can make the following conclusion.

The velocity profile increase with an increase of the free convection current

An increasing in Heat source parameter, and Thermal radiation parameter increases the velocity profile and concentration profile of the flow field at all points.

An increase in Schmidt number and permeability parameter decrease the velocity profile of the flow.

A growing Thermal radiation parameter and Heat absorption parameter decrease temperature of the flow field at all points.

The increase in Schmidt number and permeability parameter decreases the concentration of the flow field at all points.

ISSN 2229-5518

An increasing, Schmidt number, Heat source parameter, and thermal radiation parameter decreases the skin- friction coefficient.

The Schmidt number and permeability parameter decreases the Sherwood number of the flow field at all points.

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